

Roll No. ....

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BT-2/M-14

8201

**MATHEMATICS-II**

Paper-Math-102-E

Time Allowed : 3 Hours] [Maximum Marks : 100

**Note :** Attempt five questions in all, selecting at least one question from each Unit. All questions carry equal marks.

**UNIT-I**

1. (a) Find the inverse of

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

- (b) Determine the values of  $\lambda$  for which the following set of equations may possess non-trivial solution :

$$3x_1 + x_2 - \lambda x_3 = 0$$

$$4x_1 - 2x_2 - 3x_3 = 0$$

$$2\lambda x_1 + 4x_2 + \lambda x_3 = 0.$$

For each permissible value of  $\lambda$ , determine the general solution.

2. (a) State Cayley-Hamilton theorem, use it to find

$$A^8 \text{ if } A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}.$$

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- (b) If  $A = \begin{bmatrix} 2+i & 3 & -1+3i \\ -5 & i & 4-2i \end{bmatrix}$ , show that  $AA^*$  is a Hermitian matrix, where  $A^*$  is the conjugate transpose of  $A$ .

**UNIT-II**

3. (a) Solve  $2ydx + x(2 \log x - y)dy = 0$ .  
(b) Find the orthogonal trajectory of the family of curves  $r^n = a \sin n\theta$ .  
4. (a) Solve  $(D^2 - 4y + 4)y = 8x^2 e^{2x} \sin 2x$ .  
(b) Solve by the method of variation of parameters

$$\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 2y = e^x \tan x.$$

**UNIT-III**

5. (a) Find the Laplace transform :

(i)  $\left(\sqrt{t} - \frac{1}{\sqrt{t}}\right)^3$ .

(ii)  $|t-1| + |t+1|, t \geq 0$ .

- (b) Find the inverse Laplace transform of

$$\frac{s}{s^4 + 4a^2}.$$

6. (a) Solve by the use of Laplace transform

$$x \frac{d^2 y}{dx^2} + \frac{dy}{dx} + xy = 0, y(0) = 2, y'(0) = 0.$$

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- (b) Solve the following simultaneous equation by the use of Laplace transform :

$$\frac{dx}{dt} - y = et, \frac{dy}{dt} + x = \sin t, \text{ with } x(0)=1, y(0)=0.$$

#### UNIT-IV

7. (a) Solve :

$$(x-y)(px-qy)=(p-q)^2.$$

- (b) Solve :

$$2z+p^2+qy+2y^2=0.$$

8. (a) Solve :

$$\frac{\partial^3 z}{\partial x^3} - 2 \frac{\partial^3 z}{\partial x \partial y} = 2e^x + 3x^2y.$$

- (b) Solve :

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

with boundary conditions

$$u(x, 0)=3 \sin n\pi x \quad u(0,t)=0 \text{ and } u(1,t)=0,$$

where  $0 < x < 1, t > 0$ .